

# PAPR Analysis of FFT and Wavelet based OFDM Systems

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**Abstract** – OFDM is an efficient modulation technique for wideband digital communication. A major drawback of OFDM modulation is the high Peak-to-Average Power Ratio (PAPR) of the transmitted signal which limits its transmission range of communication. As PAPR increases it reduces the average power which affect on total power consumption of Transmitter. Normally, OFDM is implemented using FFT. In recent years it is observed that, OFDM can be implemented using orthogonal wavelets. All the benefits of FFT-OFDM can be achieved, if we replace traditional sinusoid carriers with suitable wavelets. SLM & PTS techniques for PAPR reduction are applied on the FFT & Wavelet based OFDM.

**Keywords** – Wavelet Based Orthogonal Frequency Division Multiplexing (WOFDM), Peak to Average Power Ratio (PAPR), Power Spectral Density (PSD).

## I. INTRODUCTION

OFDM has many well documented advantages including high data rates, high spectral efficiency; multipath delay spread tolerance, power efficiency and a very strong immunity to the frequency selective fading channels. A serial data symbol stream is multiplexed into a large number of orthogonal sub-channel makes the OFDM signals spectral bandwidth efficient. The high PAPR brings on the OFDM signal distortion in the nonlinear region of the high power amplifier (HPA) and the signal distortion effects the degradation of Bit error rate (BER)[1].

Cyclic Prefix (CP) is added to each symbol to reduce the ISI (inter-symbol interference) caused by multipath wireless channel, which affect on spectral inefficiency. Cyclic prefix also causes ripples in the power spectral density (PSD). As the Guard Interval (GI) or CPs discarded in the receiver, system throughput is greatly reduced[2]. AWGN, Rayleigh and Rician channels are going to use for the communication [5]. Rayleigh fading is a rational model when there are many objects in the environment that scatters the transmitted signal before it arrives at the receiver. Each channel will have random input data as an input and it generates the output.

Rician Fading is a non-deterministic model and it occurs when a transmitted signal accidentally cancels itself. An Additive White Gaussian Noise (AWGN) channel adds White Gaussian noise to the signal when it is passed through the channel. In the case of white Gaussian noise

the values at any pair of times are identically distributed and statistically independent on each other. Three channel outputs are compared to observe the difference in the effect of noise insertion in each channel[5].

In general, OFDM signal is a linear combination of the data-modulated subcarriers can have a very large amplitude changes which results in a large PAPR for each symbol of data which is a limitation of OFDM that tends to a nonlinear distortion in practical implementations of high power amplifier (HPA). To overcome the effect of high PAPR on transmitted data, different PAPR reduction techniques are implemented.

Some of them are clipping, filtering, coding schemes, phase optimization, nonlinear companding transforms, Tone Reservation (TR) and Tone Injection (TI)[1], constellation shaping, Partial Transmission Sequence (PTS)[4] and Selective Mapping (SLM)[6]. SLM and PTS can provide good performance for PAPR reduction, but it increases the computational complexity. Several techniques have been proposed based on low-complexity SLM and PTS techniques.

The better option to design a multicarrier modulation with a low value of PAPR, an easy solution is to use wavelets instead of orthogonal subcarriers.

## II. OFDM SIGNALS AND PAPR

### A) FFT Based OFDM:

OFDM signal is generated by an  $N$ -point Inverse Fast Fourier Transform (IFFT) in the transmitter, and the Fast Fourier Transform (FFT) is placed at the receiver to restore the signal[1]. Discrete time OFDM signal can be written as

$$X(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi}{NL}kn}, 0 \leq n \leq NL-1 \quad \dots\dots(1)$$

Where is the symbol carried by the sub-carrier,  $L$  is the oversampling factor. An OFDM signal consists of an “ $N$ ” number of independently modulated subcarriers, which can give a very large PAPR when added up coherently. PAPR is the ratio between the maximum power and the average power of the complex signal. The PAPR for the time domain OFDM signal can be defined as  $P$

$$PAPR = \left[ \frac{\max x(n)^2}{E\{x(n)^2\}} \right] \quad \dots\dots\dots(2)$$

Where  $E[\cdot]$  denotes the expectation operation. PAPR increases proportionally with the number of subcarriers. Reducing  $\max|x(n)|$  is the main goal of PAPR reduction techniques. It is easier to analyze the PAPR using CCDF, which is the most common way to evaluate the PAPR by calculating the probability of PAPR when it exceeds a certain level  $PAPR_0$ . The CCDF expression of the PAPR of OFDM signals with relatively small subcarriers  $N$  can be written as

$$CCDF = P(PAPR > PAPR_0) = 1 - (1 - \exp(-PAPR_0))^N \quad \dots\dots\dots (3)$$

This equation can be interpreted as the probability that the PAPR of a symbol block exceeds some threshold level  $PAPR_0$ .

Block diagram of FFT-OFDM is as follows.

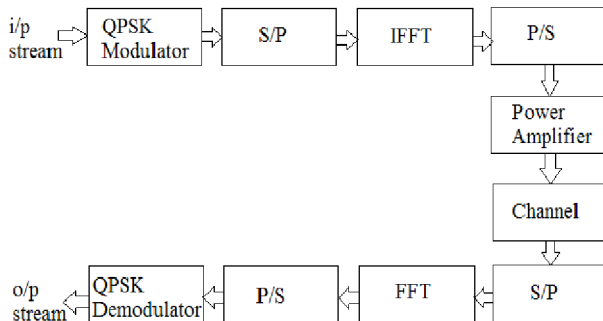


Fig.1. Block diagram of FFT based OFDM

**B) Fourier vs. Wavelet:**

In Fourier analysis a signal is located into a set of infinite sum of Sine's and Cosine's to provide the Orthogonality relationship between them. On the other hand, in wavelet transform the signal is first decomposed by a low-pass (LP) and a high-pass (HP) filter. Half of the frequency components have been filtered out at filter outputs and hence can be down-sampled. The approximation coefficients  $H\{n\}$  and Detail coefficients  $G\{n\}$  are filtered out from respective filters, which are the wavelet's half-band low pass filter and half-band high pass filter impulse responses[7].

In wavelet decomposition process, both the detail and approximation coefficients can be divided into a second level details and approximations factors. These two sets of coefficients are obtained by performing convolution between the input signals and wavelet filter coefficients. Decomposition process is repeated by a series of high-pass (HP) and low-pass (LP) filters until we obtain wavelet sequence that is orthogonal in nature. The original signal is then reconstructed by performing the reverse operation of this decomposition. Reconstruction of signal using Wavelet coefficient is more accurate than FFT coefficient.

**C) Wavelet based OFDM:**

A wavelet based OFDM system is discussed in this section. Fig. 2 shows the transceiver circuitry for Wavelet based OFDM. Wavelet transform has a higher degree of side lobe suppression and the loss of Orthogonality leads

to lesser ISI and ICI. In Wavelet OFDM the FFT and IFFT blocks are replaced by DWT and IDWT respectively[7].

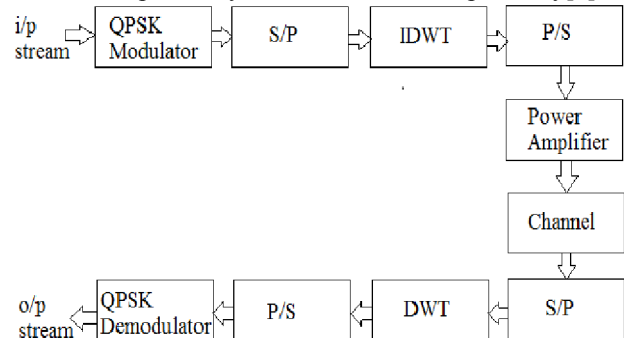


Fig.2. Block diagram of Wavelet based OFDM

It is observed that basic Wavelet transform offers lesser flexibility than the wavelet packet transform. The wavelet can be implemented using QMF filter banks" but this cannot fulfill linear phase filtering, therefore we can also use bi-orthogonal filter banks. These bi-orthogonal filters provide linear phase filtering and the design is flexible.

**Advantages of WOFDM:**

- It requires lesser overhead as it doesn't require CP.
- It doesn't require a pilot tone which takes about 8% of the sub-bands.
- DWT-OFDM is inherently robust to ISI and ICI.

**D) Channel Model for Communication**

**1) AWGN channel:**

The basis function for a bi-orthogonal wavelet is not orthogonal to each other. This causes the AWGN to become correlated within a subcarrier and thus an AWGN channel doesn't remain an AWGN channel. Thus the non-Orthogonality has now become a problem, but some wavelets are still useful. For the Haar wavelet case the performance of a DWT-OFDM is much better than the DFT-OFDM case when no timing, frequency synchronization is done [2].

**2) Flat Rayleigh fading channel:**

For a Flat Rayleigh fading channel the channel can be modeled as follows: Let the transmitted codeword be  $X$  and received be  $Y$  Then

$$Y = aX + n \quad \dots\dots\dots (4)$$

Where "a" is the fading coefficient and it is random variable for PDF.

$$f(a) = 2a e^{-a^2} \quad \dots\dots\dots (5)$$

and "n" is the AWGN[2].

For the Flat Rayleigh channel it is found that the DWT-OFDM is advantageous for some SNR ranges (SNR<25dB) while DFT-OFDM is better for some. For the bi-orthogonal wavelet packets, only a few wavelets have worse performance than the single subcarrier OFDM.

**3) Rician Fading Channel:**

The Multipath Rician Fading Channel implements a baseband simulation of a multipath Rician fading propagation channel. This is useful for modeling mobile wireless communication systems when the transmitted signal can travel to the receiver along a dominant line-of-sight or direct path. This channel accepts only frame-based complex signals at its input. The sample time is inherited from the input signal. The input signal must have a discrete sample time greater than 0. Relative motion between the transmitter and receiver causes Doppler shifts in the signal frequency[5].

**E) Reduction Techniques**

PAPR reduction methods can be classified into distortion-less and distortion techniques. Distortion techniques are considered to introduce spectral regrowth. They do not require any side information to be sent and they have low complexity compared to the distortion-less techniques. Here the simplest method is to clip the peak amplitude of the OFDM signal to some desired maximum which is an irreversible nonlinear process which surely degrades the system performance [3][4]. Distortion-less techniques on the other hand, do not suffer from spectral regrowth, but they do require sending side information to the receiver. Most of the methods in distortion-less category are based on the same idea of selecting the signal to be transmitted from a set of different representations with less PAPR which would degrade the performance of system. In this article, PAPR performances of wavelet based OFDM have been simulated for based on SLM and PTS techniques[3].

**a) Selective Mapping (SLM):**

The technique of selected mapping (SLM) for PAPR reduction was proposed in 1996. In SLM from a set of candidate signals which are generated to represent the same information, the signal with lowest PAPR is selected and transmitted. The information about this selection also needs to be explicitly transmitted along with the selected signal as side information.

SLM needs to transmit the information to the receiver, about the selected signal, as side information. If there is a error in the received side information, then the receiver cannot recover the information from the transmitted selected signal. Therefore a strong protection against transmission errors is needed regarding side information. Once the receiver has this side information then the decoding process is very simple. SLM is can be employed for larger number of sub-carriers with moderate complexity. The technique uses codes only for PAPR reduction and does not include error correction capabilities of codes. This scheme is aimed at decreasing the frequency of peak occurrence rather than elimination of peaks. The drawbacks include multiple numbers of IFFT operations leading to increased complexity and the need for transfer of side information to the receiver without any margin for transmission errors[5].

**b) Partial Transmit Sequences (PTS) :**

In partial transmit sequences (PTS), initially partitioning of the data block into non-overlapping sub-blocks is done. Then these sub-blocks are rotated with rotation factors which are statistically independent. Subsequently the information about rotation factor, which generates the lowest peak amplitude in time domain data, is transmitted to the receiver. The need for transmission of side information can also be overcome if differential modulation is employed for each sub block and block partitioning is known in the receiver[3].

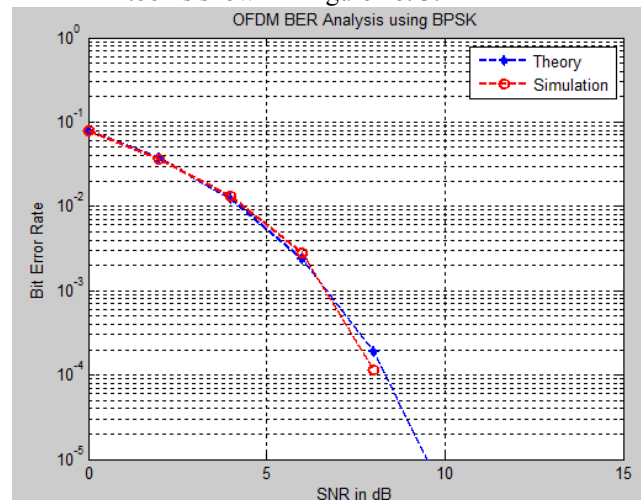
**III. SIMULATION**

Simulation Parameters for the basic BPSK modulated FFT-OFDM system using AWGN channel is shown in Table No.1.

Table 1: Simulation Parameters

SR.NO.	PARAMETERS	VALUES
1	No. of subcarriers	52
2	No. of symbols	100
3	FFT size	64
4	No. of data sub-carriers	48
5	SNR in dB	0:2:30

The simulation result for above parameters using MATLAB tool is shown in Figure no. 3.



The result of FFT-OFDM for BER Vs SNR ratio is calculated. SNR value is varies from 0 to 30 with difference of 5dB range. It is observed that as SNR increases, Bit Error rate going to reduce. Random binary input data is selected for operation.

**IV. CONCLUSION**

One of the serious drawbacks of OFDM systems is that the composite transmit signal can exhibit a very high peak

power. However, the high peak to average power ratio of an OFDM signal has limited its applicability for practical low power systems. The method makes use of the special property of discrete wavelets transmission that only a few numbers of large coefficients dominates the representation. In proposed topic the FFT-OFDM and Wavelet OFDM system get compared according to its PAPR response. SLM and PTS technique are used to compare the results.

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