

Relationship between New Types of Transitive Maps and Minimal Systems

Mohammed Nokhas Murad

University of Sulaimani
Faculty of Science and Science Education,
School of Science, Math Department
Email: muradkakaee@yahoo.com

Abstract – The concepts of topological θ -transitive maps, α -transitive maps, θ -minimal and α -minimal mappings were introduced by M. Nokhas Murad. In this paper, I study the relationship between two different notions of transitive maps, namely topological θ -transitive maps, topological α -transitive maps and investigate some of their properties in two topological spaces (X, τ) and (X, τ^α) , τ denotes the θ -topology (resp. α -topology) of a given topological space (X, τ) . The two notions are defined by using the concepts of θ -irresolute map and α -irresolute map respectively. Also, we study the relationship between two types of minimal mappings, namely, θ -minimal mapping and α -minimal mapping, and I will prove that the properties of θ -transitive, α -mixing and α -minimal maps are preserved under θ -conjugacy. The main results are the following propositions:

- 1) Every topologically θ -transitive map is a topologically α -transitive map which implies topologically θ -transitive map, but the converse not necessarily true.
- 2) Every θ -minimal map is a minimal map which implies α -minimal map in topological spaces, but the converse not necessarily true..

Keywords – Topologically θ - Transitive, α - Irresolute, θ - Transitive, α -Dense.

I. INTRODUCTION

Recently there has been some interest in the notion of a locally closed subset of a topological space. According to Bourbaki [16] a subset S of a space (X, τ) is called locally closed if it is the intersection of an open set and a closed set. Ganster and Reilly used locally closed sets in [13] and [14] to define the concept of LC-continuity, i.e. a function $f : (X, \tau) \rightarrow (X, \sigma)$ is LC-continuous if the inverse with respect to f of any open set in Y is closed in X . The study of semi open sets and semi continuity in topological spaces was initiated by Levine [6]. Bhattacharya and Lahiri [8] introduced the concept of semi generalized closed sets in topological spaces analogous to generalized closed sets which was introduced by Levine [5]. Throughout this paper, the word "space" will mean topological space. The collections of semi-open, semi-closed sets and θ -sets in (X, τ) will be denoted by $SO(X, \tau)$, $SC(X, \tau)$ and τ^α respectively. Njastad [7] has shown that τ^α is a topology on X with the following properties: $\tau \subseteq \tau^\alpha$, $(\tau^\alpha)^\alpha = \tau^\alpha$ and $S \in \tau^\alpha$ if and only if $S = U \setminus N$ where $U \in \tau$ and N is nowhere dense (i.e. $\text{Int}(\text{Cl}(N)) = \emptyset$) in (X, τ) . Hence $\tau = \tau^\alpha$ if and only

if every nowhere dense (nwd) set in (X, τ) is closed, therefore every transitive map implies θ -transitive. Also if every θ -open set is locally closed then every transitive map implies θ -transitive; and this structure also occurs if (X, τ) is locally compact and Hausdorff [36, p. 140, Ex. B]] and every θ -open set is locally compact, then every θ -open set is locally closed. Clearly every θ -set is semi-open and every nwd set in (X, τ) is semi-closed. Andrijevic [1] has observed that $SO(X, \tau^\alpha) = SO(X, \tau)$, and that $N \subseteq X$ is nwd in (X, τ^α) if and only if N is nwd in (X, τ) .

In 1943, Fomin [27] introduced the notion of θ -continuous maps. The notions of θ -open sets, θ -closed sets and θ -closure were introduced by Velićko [19] for the purpose of studying the important class of H -closed spaces in terms of arbitrary fiber-bases. Dickman and Porter [20], [21], Joseph [22] and Long and Herrington [31] continued the work of Velićko. We introduce the notions of θ -type transitive maps, θ -minimal maps and show that some of their properties are analogous to those for topologically transitive maps. Also, we give some additional properties of θ -irresolute maps. We denote the interior and the closure of a subset A of X by $\text{Int}(A)$ and $\text{Cl}(A)$, respectively. By a space X , we mean a topological space (X, τ) . A point $x \in X$ is called a θ -adherent point of A [19], if $A \cap \text{Cl}_\theta(V) \neq \emptyset$ for every open set V containing x . The set of all θ -adherent points of a subset A of X is called the θ -closure of A and is denoted by $\text{Cl}_\theta(A)$.

A subset A of X is called θ -closed if $A = \text{Cl}_\theta(A)$.

Dontchev and Maki [22] have shown that if A and B are subsets of a space (X, τ) , then

$$\text{Cl}_\theta(A \cup B) = \text{Cl}_\theta(A) \cup \text{Cl}_\theta(B) \text{ also } \text{Cl}_\theta(A \cap B) = \text{Cl}_\theta(A) \cap \text{Cl}_\theta(B)$$

Note also that the θ -closure of a given set need not be a θ -closed set. But it is always closed. Dickman and Porter [20] proved that a compact subspace of a Hausdorff space is θ -closed. Moreover, they showed that a θ -closed subspace of a Hausdorff space is closed. Jankovič [25] proved that a space (X, τ) is Hausdorff if and only if every compact set is θ -closed. The complement of a θ -closed set is called a θ -open set. The family of all θ -open sets forms a topology on X and is denoted by τ^θ or θ -topology. This topology is coarser than τ and that a space (X, τ) is regular if and only if $\tau = \tau^\theta$ [26]. Then we observe that every θ -type transitive maps is transitive if (X, τ) is regular. In general, $\text{Cl}_\theta(A)$ will not be the closure of A with

respect to (X, τ^θ) . It is easily seen that one always has $A \subseteq Cl(A) \subseteq Cl_\theta(A) \subseteq Cl_\theta(A) \subseteq \bar{A}^\theta$ where \bar{A}^θ denotes the closure of A with respect to (X, τ^θ) . It is also obvious that a set A is θ -closed in (X, τ) if and only if it is closed in (X, τ^θ) . The space (X, τ^θ) is called sometimes the semi regularization of (X, τ) . A function $f : X \rightarrow Y$ is closure continuous [29] (θ -continuous) at $x \in X$ if given any open set V in Y containing $f(x)$, there exists an open set U in X containing x such that $f(Cl(U)) \subseteq Cl(V)$. [29] In this paper, we will study the relationship between new classes of topological transitive maps called α -type transitive and θ -type transitive and also, new classes of α -minimal maps and θ -minimal maps. We have shown that every θ -transitive map is a α -type transitive map, but the converse not necessarily true and that every θ -minimal map is a α -minimal map, but the converse not necessarily true we will also study some of their properties.

II. PRELIMINARIES AND DEFINITIONS

In this section, we recall some of the basic definitions. Let X be a space and $A \subset X$. The intersection (resp. closure) of A is denoted by $Int(A)$ (resp. $Cl(A)$).

Definition 2.1 [6] A subset A of a topological space X will be termed semi- open (written S.O.) if and only if there exists an open set U such that $U \subset A \subset Cl(U)$.

Definition 2.2 [8] Let A be a subset of a space X then semi closure of A defined as the intersection of all semi - closed sets containing A is denoted by $sClA$.

Definition 2.3 [9] Let (X, τ) be a topological space and an operator from τ to (X) i.e $\tau^\alpha : (X)$, where (X) is a power set of X. We say that τ^α is an operator associated with τ if $U \subset \tau^\alpha(U)$ for all U

Definition 2.4 [10] Let (X, τ) be a topological space and τ^α an operator associated with τ . A subset A of X is said to be τ^α -open if for each $x \in X$ there exists an open set U containing x such that $\tau^\alpha(U) \subset A$. Let us denote the collection of all τ^α -open, semi-open sets in the topological space (X, τ) by τ^α , $SO(\tau^\alpha)$, respectively. We then have $\tau \subseteq \tau^\alpha \subseteq SO(\tau)$. A subset B of X is said to be τ^α -closed [7] if its complement is τ^α -open.

Definition 2.5 [9] Let (X, τ) be a space. An operator τ^α is said to be regular if, for every open neighborhoods U and V of each $x \in X$, there exists a neighborhood W of x such that $\tau^\alpha(W) \subset U \subset V$. Note that the family τ^α of τ^α -open sets in (X, τ) always forms a topology on X, when τ^α is considered to be regular finer than τ .

Theorem 2.6 [30] For subsets A, B of a space X, the following statements hold:

- (1) $D(A) \subset D_\theta(A)$, where $D(A)$ is the derived set of A
- (2) If $A \subset B$, then $D_\theta(A) \subset D_\theta(B)$
- (3) $D_\theta(A) \cup D_\theta(B) = D_\theta(A \cup B)$.

and $D_\theta(A \cap B) \subset D_\theta(A) \cap D_\theta(B)$

Note that the family τ^θ of θ -open sets in (X, τ) always forms a topology on X denoted τ^θ -topology and that τ^θ -topology coarser than τ .

Definition 2.7 [4]: Let A be a subset of a space X. A point x is said to be an α -limit point of A if for each α -open U containing x, $U \cap (A \setminus x) \neq \emptyset$. The set of all α -limit points of A is called the α -derived set of A and is denoted by $D_\alpha(A)$.

Definition 2.8 [4] For subsets A and B of a space X, the following statements hold true:

- 1) $D_\alpha(A) \subset D(A)$ where $D(A)$ is the derived set of A
- 2) if $A \subset B$ then $D_\alpha(A) \subset D_\alpha(B)$
- 3) $D_\alpha(A) \cup D_\alpha(B) \subset D_\alpha(A \cup B)$
- 4) $D_\alpha(A \cup D_\alpha(A)) \subset A \cup D_\alpha(A)$.

Definition 2.9 [10]: The point x $\in X$ is in the α -closure of a set $A \subset X$ if $x \in \bigcap \{U \mid A \subset U\}$, for each open set U containing x. The α -closure of a set A is the intersection of all α -closed sets containing A and is denoted by $Cl_\alpha(A)$.

Remark 2.10: For any subset A of the space X, $A \subset Cl(A) \subset Cl_\alpha(A)$

Definition 2.11 [10] Let (X, τ) be a topological space. We say that a subset A of X is α -compact if for every α -open covering \mathcal{C} of A there exists a finite sub-collection $\{C_1, C_2, \dots, C_n\}$ of \mathcal{C} such that $A \subset \bigcup_{i=1}^n C_i$.

Properties of α -compact spaces have been investigated by Rosa, E etc. and Kasahara, S [9,10]. The following results were given by Rosas, E etc. [9].

Theorem 2.12 Let (X, τ) be a topological space and τ^α an operator associated with τ . $A \subset X$ and $K \subset A$. If A is τ^α -compact and K is τ^α -closed then K is τ^α -compact.

Theorem 2.13 Let (X, τ) be a topological space and τ^α be a regular operator on τ . If X is τ^α - T_2 (see Rosa, E etc. and Kasahara, S) [9,10] and $K \subset X$ is τ^α -compact then K is τ^α -closed.

Definition 2.14 [10] The intersection of all τ^α -closed sets containing A is called τ^α -closure of A, denoted by $Cl_\alpha(A)$.

Remark 2.15 For any subset A of the space X, $A \subset Cl(A) \subset Cl_\alpha(A)$.

Lemma 2.16 For subsets A and A_i ($i \in I$) of a space (X, τ) , the following hold:

- 1) $A \subset Cl_\alpha(A)$
- 2) $Cl_\alpha(A)$ is closed; $Cl_\alpha(Cl_\alpha(A)) = Cl_\alpha(A)$
- 3) If $A \subset B$ then $Cl_\alpha(A) \subset Cl_\alpha(B)$
- 4) $Cl_\alpha(\bigcap \{A_i : i \in I\}) \subset \bigcap \{Cl_\alpha(A) : i \in I\}$
- 5) $Cl_\alpha(\bigcup \{A_i : i \in I\}) = \bigcup \{Cl_\alpha(A) : i \in I\}$

Lemma 2.17 The collection of τ^α -compact subsets of X is closed under finite unions. If τ^α is a regular operator and

X is an $-T_2$ space then it is closed under arbitrary intersection.

Definition 2.18 Let (X, τ) be a topological space, A a subset of X . The $\text{int}_\alpha(A) = \cup\{U : U \text{ is } \alpha\text{-open and } U \subset A\}$.

Remark 2.19 A subset A is α -open if and only if $\text{int}_\alpha(A) = A$.

Proof: The proof is obvious from the definition.

Definition 2.20 Let (X, τ) and (Y, σ) be two topological spaces, a map $f : X \rightarrow Y$ is said to be α -continuous if for each open set H of Y , $f^{-1}(H)$ is α -open in X .

Theorem 2.21 [4]: For any subset A of a space X , $Cl_\alpha(A) = A \cup Cl_\alpha(A)$.

Theorem 2.22. [4]: For subsets A, B of a space X , the following statements are true:

- 1) $\text{int}_\alpha(A)$ is the largest α -open contained in A
- 2) $\text{int}_\alpha(\text{int}_\alpha(A)) = \text{int}_\alpha(A)$
- 3) If $A \subset B$ then $\text{int}_\alpha(A) \subset \text{int}_\alpha(B)$
- 4) $\text{int}_\alpha(A) \cup \text{int}_\alpha(B) \subset \text{int}_\alpha(A \cup B)$
- 5) $\text{int}_\alpha(A) \cap \text{int}_\alpha(B) \supset \text{int}_\alpha(A \cap B)$

Lemma 2.23 [7] For any α -open set A and any α -closed set C , we have

- 1) $Cl_\alpha(A) = Cl(A)$
- 2) $\text{int}_\alpha(C) = \text{int}(C)$
- 3) $\text{int}_\alpha(Cl_\alpha(A)) = \text{int}(Cl(A))$

Remark 2.24. [4]: It is not always true that every α -open set is an open set, as shown in the following example:

Example 2.25 Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, \{c, d\}, X\}$. Hence $\tau_\alpha = \{\emptyset, \{c, d\}, \{b, c, d\}, \{a, c, d\}, X\}$. So $\{b, c, d\}$ is α -open but not open.

Theorem 2.26 [30] For subsets A, B of a topological space X , the following statements are true:

- (1) $\text{Int}_\theta(A)$ is the union of all open sets of X whose closures are contained in A
- (2) A is θ -open $\Leftrightarrow \text{Int}_\theta(A) = A$.
- (3) $\text{Int}_\theta(\text{Int}_\theta(A)) \subset \text{Int}_\theta(A)$.
- (4) $X \setminus \text{Int}_\theta(A) = Cl_\theta(X \setminus A)$.
- (5) $X \setminus Cl_\theta(A) = \text{Int}_\theta(X \setminus A)$.
- (6) If $A \subset B$ then $\text{Int}_\theta(A) \subset \text{Int}_\theta(B)$.
- (7) $\text{Int}_\theta(A) \cup \text{Int}_\theta(B) \subset \text{Int}_\theta(A \cup B)$
- (8) $\text{Int}_\theta(A) \cap \text{Int}_\theta(B) = \text{Int}_\theta(A \cap B)$.

III. TRANSITIVE AND MINIMAL SYSTEMS

Topological transitivity is a global characteristic of dynamical systems. By a dynamical system (X, f) [15] we mean a topological space X together with a continuous map $f : X \rightarrow X$. The space X is sometimes called the phase space of the system. A set $A \subseteq X$ is called f -invariant if $f(A) \subseteq A$.

A topological system (X, f) is called *minimal* if X does not contain any non-empty, proper, closed f -invariant subset. In such a case we also say that the map f itself is minimal. Thus, one cannot simplify the study of the dynamics of a minimal system by finding its nontrivial closed subsystems and studying first the dynamics restricted to them.

Given a point x in X , $O_f(x) = \{x, f(x), f^2(x), \dots\}$ denotes its orbit (by an orbit we mean a forward orbit even if f is a homeomorphism) and $\omega_f(x)$ denotes its α -limit set, i.e. the set of limit points of the sequence $x, f(x), f^2(x), \dots$. The following conditions are equivalent:

- (X, f) is α -minimal (resp. α -minimal),
- every orbit is α -dense (resp. α -dense) in X ,
- $\omega_f(x) = X$ for every $x \in X$.

A minimal map f is necessarily surjective if X is assumed to be Hausdorff and compact.

Now, we will study the Existence of minimal sets. Given a dynamical system (X, f) , a set $A \subseteq X$ is called a *minimal set* if it is non-empty, closed and invariant and if no proper subset of A has these three properties. So, $A \subseteq X$ is a minimal set if and only if $(A, f|_A)$ is a minimal system. A system (X, f) is minimal if and only if X is a minimal set in (X, f) .

The basic fact discovered by G. D. Birkhoff is that in any compact system (X, f) there are minimal sets. This follows immediately from the Zorn's lemma. Since any orbit closure is invariant, we get that *any compact orbit closure contains a minimal set*. This is how compact minimal sets may appear in non-compact spaces. Two minimal sets in (X, f) either are disjoint or coincide. A minimal set A is strongly f -invariant, i.e. $f(A) = A$. Provided it is compact Hausdorff

Let (X, f) be a topological system, and $f : X \rightarrow X$ a homeomorphism of X onto itself. For A and B subsets of X , we let $N(A, B) = \{n \in \mathbf{Z} : f^n(A) \cap B \neq \emptyset\}$

We write $N(A, B) = N(x, B)$ for a singleton $A = \{x\}$ thus $N(x, B) = \{n \in \mathbf{Z} : f^n(x) \in B\}$

For a point $x \in X$ we write $O_f(x) = \{f^n(x) : n \in \mathbf{Z}\}$ for the orbit of x and $Cl_\alpha(O_f(x))$ for the α -closure of $O_f(x)$. We say that the topological system (X, f) is α -type point transitive if there is a point $x \in X$ with $O_f(x)$ α -dense. Such a point is called α -type transitive. We say that the topological system (X, f) is topologically α -type transitive (or just α -type transitive) if the set $N(U, V)$ is nonempty for every pair U and V of nonempty α -open subsets of X .

3.1 Topologically α -Transitive Maps:

In [35], we introduced and defined a new class of transitive maps that are called topologically α -transitive

maps on a topological space (X, τ) , and we studied some of their properties and proved some results associated with these new definitions. We also defined and introduced a new class of α -minimal maps. In this paper we discuss the relationship between topologically α -transitive maps and α -transitive maps. On the other hand, we discuss the relationship between α -minimal and α -minimal in topological systems.

Definition 3.1.1 Let (X, τ) be a topological space. A subset A of X is called α -dense in X if $Cl_\alpha(A) = X$.

Remark 3.1.2 Any α -dense subset in X intersects any α -open set in X .

Proof: Let A be an α -dense subset in X , then by definition, $Cl_\alpha(A) = X$, and let U be a non-empty α -open set in X . Suppose that $A \cap U = \emptyset$. Therefore $B = U^c$ is α -closed and $A \subset U^c = B$. So $Cl_\alpha(A) \subset Cl_\alpha(B)$, i.e. $Cl_\alpha(A) \subset B$, but $Cl_\alpha(A) = X$, so $X \subset B$, this contradicts that $U \cap A = \emptyset$.

Definition 3.1.3 [12] A map $f : X \rightarrow Y$ is called α -irresolute if for every α -open set H of Y , $f^{-1}(H)$ is α -open in X .

Example 3.1.4 [35] Let (X, τ) be a topological space such that $X = \{a, b, c, d\}$ and $\tau = \{ \emptyset, X, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\} \}$ and the set of all α -closed sets is $C(X, \tau) = \{ \emptyset, X, \{c, d\}, \{a, c, d\}, \{a, d\}, \{a, c\}, \{d\}, \{c\} \}$. Then define the map $f : X \rightarrow X$ as follows $f(a) = a, f(b) = b, f(c) = d, f(d) = c$, we have f is α -irresolute because $\{b\}$ is α -open and $f^{-1}(\{b\}) = \{b\}$ is α -open; $\{a, b\}$ is α -open and $f^{-1}(\{a, b\}) = \{a, b\}$ is α -open; $\{b, c\}$ is α -open and $f^{-1}(\{b, c\}) = \{b, d\}$ is α -open; $\{a, b, c\}$ is α -open and $f^{-1}(\{a, b, c\}) = \{a, b, d\}$ is α -open; $\{a, b, d\}$ is α -open and $f^{-1}(\{a, b, d\}) = \{a, b, c\}$ is α -open so f is α -irresolute.

Definition 3.1.5 A subset A of a topological space (X, τ) is said to be nowhere α -dense, if its α -closure has an empty α -interior, that is, $int_\alpha(Cl_\alpha(A)) = \emptyset$.

Definition 3.1.6 [35] Let (X, τ) be a topological space, $f : X \rightarrow X$ be α -irresolute map then f is said to be topological α -transitive if every pair of non-empty α -open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \emptyset$. In the forgoing example 3.1.4: we have f is α -transitive because b belongs to any non-empty α -open set V and also belongs to $f(U)$ for any α -open set it means that $f(U) \cap V \neq \emptyset$ so f is α -transitive.

Example 3.1.7 Let (X, τ) be a topological space such that $X = \{a, b, c\}$ and $\tau = \{ \emptyset, \{a\}, X \}$. Then the set of all α -open sets is $\tau_\alpha = \{ \emptyset, \{a\}, \{a, b\}, \{a, c\}, X \}$. Define $f : X \rightarrow X$ as follows $f(a) = b, f(b) = b, f(c) = c$. Clearly f is continuous because $\{a\}$ is open and $f(\{a\}) = \{b\}$ is open. Note that f is transitive because $f(\{a\}) = \{b\}$ implies that $f(\{a\}) \cap \{b\} \neq \emptyset$. But f is not α -transitive because for each n in \mathbb{N} , $f^n(\{a\}) \cap \{a, c\} = \emptyset$; since $f^n(\{a\}) = \{b\}$ for every n in \mathbb{N} , and $\{b\} \cap \{a, c\} = \emptyset$. So we have f is not α -transitive, so we show that transitivity not implies α -transitivity.

Definition 3.1.8 Let (X, τ) be a topological space. A subset A of X is called α -dense in X if $Cl_\alpha(A) = X$.

Remark 3.1.9 Any α -dense subset in X intersects any α -open set in X .

Proof: Let A be a α -dense subset in X , then by definition, $Cl_\alpha(A) = X$, and let U be a non-empty α -open set in X . Suppose that $A \cap U = \emptyset$. Therefore $B = U^c$ is α -closed because B is the complement of α -open and $A \subset U^c = B$. So $Cl_\alpha(A) \subset Cl_\alpha(B)$, i.e. $Cl_\alpha(A) \subset B$, but $Cl_\alpha(A) = X$, so $X \subset B$, this contradicts that $U \cap A = \emptyset$.

Definition 3.1.10[33] A function $f : X \rightarrow X$ is called α -irresolute if the inverse image of each α -open set is a α -open set in X .

Definition 3.1.11 A subset A of a topological space (X, τ) is said to be nowhere α -dense, if its α -closure has an empty α -interior, that is, $int_\alpha(Cl_\alpha(A)) = \emptyset$.

Definition 3.1.12 [34] Let (X, τ) be a topological space, and $f : X \rightarrow X$ α -irresolute map, then f is said to be topologically α -type transitive map if for every pair of α -open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \emptyset$.

Associated with this new definition we can prove the following new theorem.

Theorem 3.1.13 [35]: Let (X, τ) be a topological space and $f : X \rightarrow X$ be α -irresolute map. Then the following statements are equivalent:

- (1) f is topological α -transitive map
- (2) For every nonempty α -open set U in X , $\bigcup_{n=0}^{\infty} f^n(U)$ is α -dense in X
- (3) For every nonempty α -open set U in X , $\bigcup_{n=0}^{\infty} f^{-n}(U)$ is α -dense in X
- (4) If $B \subset X$ is α -closed and B is f -invariant i.e. $f(B) \subset B$. then $B = X$ or B is nowhere α -dense.
- (5) If U is α -open and $f^{-1}(U) \subset U$ then U is either empty set or α -dense in X .

Theorem 3.1.14: [34] Let (X, τ) be a topological space and $f : X \rightarrow X$ be α -irresolute map. Then the following statements are equivalent:

- (1) f is α -type transitive map
- (2) $\bigcup_{n=0}^{\infty} f^n(D)$ is α -dense in X , with D is α -open set in X .
- (3) $\bigcup_{n=0}^{\infty} f^{-n}(D)$ is α -dense in X with D is α -open set in X
- (4) If $B \subset X$ is α -closed and $f(B) \subset B$. then $B = X$ or B is nowhere α -dense
- (5) If $f^{-1}(D) \subset D$ and D is α -open in X then $D = X$ or D is α -dense in X .

IV. A-MINIMAL FUNCTIONS

We introduced a new definition on θ -minimal [35] (resp. θ -minimal [34]) maps and we studied some new theorems associated with these definitions.

Given a topological space X , we ask whether there exists θ -irresolute (resp. θ -irresolute) map on X such that the set $\{f^n(x) : n \geq 0\}$, called the orbit of x and denoted by $O_f(x)$, is θ -dense (resp. θ -dense) in X for each $x \in X$.

A partial answer will be given in this section. Let us begin with a new definition.

Definition 4.1 (θ -minimal) Let X be a topological space and f be θ -irresolute map on X with θ -regular operator associated with the topology on X . Then the dynamical system (X, f) is called θ -minimal system (or f is called θ -minimal map on X) if one of the three equivalent conditions [35] hold:

- 1) The orbit of each point of X is θ -dense in X .
- 2) $Cl_\alpha(O_f(x)) = X$ for each $x \in X$
- 3) Given $x \in X$ and a nonempty θ -open U in X , there exists $n \in \mathbb{N}$ such that $f^n(x) \in U$

Theorem 4.2 [35] For (X, f) the following statements are equivalent:

- (1) f is an θ -minimal map.
- (2) If E is an θ -closed subset of X with $f(E) \subset E$, we say E is invariant. Then $E = \emptyset$ or $E = X$.
- (3) If U is a nonempty θ -open subset of X , then $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

V. TOPOLOGICAL SYSTEMS AND CONJUGACY

In this section, I introduce and define r -conjugated topological systems (X, f) and (Y, g) , where X and Y are almost regular topological spaces. First I will define r -homeomorphism and then I will prove new theorem associated with these new definitions:

Definition 5.1 A map $h: Y \rightarrow X$ is said to be θr -homeomorphism if h is bijective and thus invertible and both h and h^{-1} are r -irresolute

Definition 5.2 Two topological systems (X, f) and (Y, g) are said to be almost regular systems if X and Y are almost regular topological spaces.

Definition 5.2 Let (X, f) and (Y, g) be two almost regular systems, then $f: X \rightarrow X$ and $g: Y \rightarrow Y$ are said to be topologically r -conjugate if there is r -homeomorphism $h: X \rightarrow Y$ such that $h \circ f = g \circ h$. We will call h a topological r -conjugacy. Thus, the two almost regular topological systems with their respective function acting on them share the same dynamics

Associated with these definitions I proceed to prove the following theorem:

Theorem 5.4 Let (X, f) and (Y, g) be two almost regular systems, if $f: X \rightarrow X$ and $g: Y \rightarrow Y$ are topologically r -conjugate. Then

- (1) f is topologically θ -transitive if and only if g is topologically θ -transitive;
- (2) f is θ -minimal if and only if g is θ -minimal;
- (3) f is topologically θ -mixing if and only if g is topologically θ -mixing.

Proof (1)

Assume that $f: X \rightarrow X$ and $g: Y \rightarrow Y$ are almost regular topological systems which are topologically r -conjugated by $h: X \rightarrow Y$. Suppose f is θ -type transitive. Let A, B be θ -open subsets of the topological space Y (to show $g^n(A) \cap B \neq \emptyset$ for some $n > 0$).

$U = h^{-1}(A)$ and $V = h^{-1}(B)$ are θ -open subsets of X since h is an θ -irresolute

Then there exists some $n > 0$ such that $f^n(U) \cap V \neq \emptyset$ since f is θ -type transitive. Thus (as $f \circ h^{-1} = h^{-1} \circ g$ implies $f^n \circ h^{-1} = h^{-1} \circ g^n$),

$\emptyset \neq f^n(h^{-1}(A)) \cap h^{-1}(B) = h^{-1}(g^n(A)) \cap h^{-1}(B)$ since h^{-1} is invertible

Therefore, $h^{-1}(g^n(A) \cap B) \neq \emptyset$ implies $g^n(A) \cap B \neq \emptyset$

Proof (2)

Assume that $f: X \rightarrow X$ and $g: Y \rightarrow Y$ are almost regular topological systems which are topologically r -conjugated by $h: Y \rightarrow X$. Thus, h is r -homeomorphism (that is, h is bijective and thus invertible and both h and h^{-1} are θ -irresolute) and $h \circ g = f \circ h$, that is, the following diagram commutes:

$$\begin{array}{ccc} Y & \xrightarrow{g} & Y \\ h \downarrow & & \downarrow h \\ X & \xrightarrow{f} & X \end{array}$$

We show that if g is θ -minimal, then f is θ -minimal. We want to show that for any $x \in X$, $O_f(x)$ is θ -dense. Since h is surjective, there exists $y \in Y$ such that $y = h^{-1}(x)$. Since g is θ -minimal, $O_g(y)$ is θ -dense.

For any non-empty θ -open subset U of X , $h^{-1}(U)$ is an θ -open set in Y since h^{-1} is θ -irresolute because h is an r -homeomorphism and it is non-empty since h is invertible. By θ -density of $O_g(y)$ there exist $k \in \mathbb{N}$ such that $g^k(y) \in h^{-1}(U) \Leftrightarrow h(g^k(y)) \in U$

Since h is r -conjugacy; as $f \circ h = h \circ g$ implies $f^k \circ h = h \circ g^k$ so $f^k(h(y)) = h(g^k(y)) \in U$ thus

$O_f(h(y))$ intersects U . This holds for any non-empty θ -open set U and thus shows that $O_f(x) = O_f(h(y))$ is θ -dense

Proof (3)

We only prove that if g is topologically α -mixing then f is also topologically α -mixing. Let U, V be two α -open subsets of X . We have to show that there is $N > 0$ such that for any $n > N$, $f^n(U) \cap V \neq \emptyset$.

$h^{-1}(U)$ and $h^{-1}(V)$ are two α -open sets since h is α -irresolute. If g is topologically α -mixing then there is $N > 0$ such that for any $n > N$, $g^n(h^{-1}(U)) \cap h^{-1}(V) \neq \emptyset$. Therefore there exists $x \in g^n(h^{-1}(U)) \cap h^{-1}(V)$. That is, $x \in g^n(h^{-1}(U))$ and $x \in h^{-1}(V)$ if and only if $x = g^n(y)$ for $y \in h^{-1}(U)$ and $h(x) \in V$.

Thus, since $h \circ g^n = f^n \circ h$, so that, $h(x) = h(g^n(y)) = f^n(h(y)) \in f^n(U)$ and we have $h(x) \in V$ that is So, $f^n(U) \cap V \neq \emptyset$. f is α -mixing.

VI. CONCLUSION

We have the following results

Proposition 6.1. Every topologically α -transitive map is a topologically transitive map which implies topologically α -transitive map, but the converse not necessarily true..

Proposition 6.2. Every α -minimal map is a minimal map which implies α -minimal map, but the converse not necessarily true..

Theorem 6.3 Let (X, f) and (Y, g) be two almost regular systems, if $f : X \rightarrow X$ and $g : Y \rightarrow Y$ are topologically α -conjugate. Then

- (1) f is topologically α -transitive if and only if g is topologically α -transitive;
- (2) f is α -minimal if and only if g is α -minimal;
- (3) f is topologically α -mixing if and only if g is topologically α -mixing

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AUTHOR’S PROFILE



Mohammed Nokhas Murad Kaki

is Assistant Prof. in Pure Math. He is interested in Topological Dynamical Systems , Catastrophe Theory and Sustainable Development in Mathematical Science. He has also executed a number of Scientific evaluation for Ph. D and MS.C Thesis

Education:

- o B.Sc. in Mathematics, Mathematics Department, College of Science-University of Al- Mustansirya in 1975
- o M.Sc. in Mathematics, Mathematics Department, College of Science, University of Baghdad in 1985.
- o Assistant Professor since 1998.

Employment

- o Mathematics Department-College of Science University of Sulaimani 20 / 8 / 2005.

Experience

- o Membership of the Scientific Council of College of Science - University of Sulaimani (2005-).
- o I have taught several topics in mathematics during 1989 till now

I: Subjects which have been taught to undergraduate (B.Sc.) students:

- o General Topology
- o Topological Dynamical Systems
- o Mathematical Analysis.
- o Foundation and Discrete of Mathematics.
- o Special Functions
- o Mathematical Transformations

II: Subjects which have been taught to graduate (M. Sc.)students:

- o Topological Transitive
- o Measure theory
- o Topological Dynamical System

M .Sc. Thesis Title

“On The Treatment of instability of phenomenon by methods of Catastrophe Theory”

Papers and Conferences

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